

BondGraph BG_nonlinear Documentation

This documentation is part of the open source BondGraph library for Modelica, published under GNU Lesser General Public License. Within this document, the models contained in the BG_nonlinear package are specified.

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Hydraulics

Sources

HSe_acc



Icon HSe_acc

Classes

| Name | Type | Description |
|--------|-----------|---|
| port_p | connector | port for flow and effort |
| port_n | connector | port for flow and effort |
| hprop | model | properties of fluid, model of viscosity and density |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|------|--------------------------|---|
| temp | parameter | t | Real | K | temperature |
| part_mass_air | parameter | - | Real | - | mass proportion of undissolved air to total mass of mixture |
| e_nom | parameter | - | Real | Pa | nominal effort = nominal pressure |
| f_nom | parameter | - | Real | m^3 / s | nominal flow = nominal volume flow |
| a | parameter | a | Real | m / s^2 | acceleration |
| d | parameter | d | Real | m | distance in direction of acceleration |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m^3 / s | flow = volume flow |
| rho | variable | ρ | Real | kg / m^3 | fluid density |
| eta | variable | η | Real | $\text{kg} / \text{s m}$ | fluid viscosity |

Mathematical description

Equation for pressure: $p = \rho \cdot a \cdot d$

Physical effect description

The HSe_acc element is a hydraulic source of effort. It describes the incurrence of pressure due to the density of fluid, distance and acceleration [1].

Hydraulics

Sources

HSe_ind

Icon HSe_ind



Classes

| Name | Type | Description |
|--------|-----------|---|
| port_p | connector | port for flow and effort |
| port_n | connector | port for flow and effort |
| hprop | model | properties of fluid, model of viscosity and density |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|------|----------|---|
| temp | parameter | t | Real | K | temperature |
| part_mass_air | parameter | - | Real | - | mass proportion of undissolved air to total mass of mixture |
| e_nom | parameter | - | Real | Pa | nominal effort = nominal pressure |
| f_nom | parameter | - | Real | m³ / s | nominal flow = nominal volume flow |
| a_1 | parameter | a_1 | Real | m | inlet cross section |
| a_2 | parameter | a_2 | Real | m | outlet cross section |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m³ / s | flow = volume flow |
| rho | variable | ρ | Real | kg / m³ | fluid density |
| eta | variable | η | Real | kg / s m | fluid viscosity |

Mathematical description

Equation dynamic pressure:

$$p = \frac{1}{2} \cdot \rho \cdot \left(\frac{f}{a}\right)^2$$

Simplified Bernoulli equation:

$$\frac{1}{2} \cdot \rho \cdot \left(\frac{f}{a_1}\right)^2 + p_1 = \frac{1}{2} \cdot \rho \cdot \left(\frac{f}{a_2}\right)^2 + p_2 = const.$$

$$\Leftrightarrow \Delta p = \frac{1}{2} \cdot \rho \cdot f^2 \cdot (a_1^{-2} - a_2^{-2})$$

Physical effect description

The HSe_ind element is a hydraulic source of effort. It is based on the Bernoulli's equation which describes the conservation of energy for incompressible fluids. It describes the change of pressure due to a change of the pipe cross section [2].

Hydraulics

Resistance

HR



Icon HR

Classes

| Name | Type | Description |
|--------|-----------|---|
| port_p | connector | port for flow and effort |
| port_n | connector | port for flow and effort |
| hprop | model | properties of fluid, model of viscosity and density |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|---------|-----------------------------------|---|
| temp | parameter | t | Real | K | temperature |
| part_mass_air | parameter | - | Real | - | mass proportion of undissolved air to total mass of mixture |
| e_nom | parameter | - | Real | Pa | nominal effort = nominal pressure |
| f_nom | parameter | - | Real | m^3 / s | nominal flow = nominal volume flow |
| a | parameter | a | Real | m | cross sectional area of pipe |
| l | parameter | l | Real | m | length of pipe |
| d_h | parameter | d_h | Real | m | hydraulic diameter of pipe |
| r_h | parameter | r_h | Real | - | relative hydraulic roughness of pipe k/d_h |
| re_crit | parameter | Re_{crit} | Real | - | critical Reynolds number |
| re_min | parameter | Re_{min} | Real | - | minimal Reynolds number |
| re_range | parameter | Re_{range} | Real | - | Reynolds number range for laminar-turbulent transition |
| r_min | parameter | - | Real | $\text{kg} / \text{m}^4 \text{s}$ | minimal resistance |
| r_max | parameter | - | Real | $\text{kg} / \text{m}^4 \text{s}$ | maximal resistance |
| par_caus | parameter | - | Integer | - | constitutive equation causality, par_caus = 1(effort out), par_caus = 2(flow out) |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m^3 / s | flow = volume flow |
| rho | variable | ρ | Real | kg / m^3 | fluid density |
| eta | variable | η | Real | $\text{kg} / \text{s m}$ | fluid viscosity |
| re | variable | Re | Real | - | reynolds number |
| lambda | variable | λ | Real | - | hydraulic pipe friction factor |

Mathematical description

Darcy-Weissbach equation:

$$p = \lambda \cdot \rho \cdot \frac{l}{d_h} \cdot \frac{|f|}{2 \cdot a^2} \cdot f$$

Equation for λ :

$$\lambda = \frac{\lambda_l}{1+exp\left(\frac{Re-Re_{crit}}{0.228\cdot Re_{range}}\right)} + \frac{\lambda_t}{1+exp\left(\frac{Re-Re_{crit}}{-0.228\cdot Re_{range}}\right)}$$

$$\lambda_l = \frac{64}{Re+Re_{min}\cdot\left(1-tanh\left(\frac{Re}{Re_{min}}\right)\right)}$$

$$\lambda_t = \left(\frac{-1.8}{\ln(10)} \cdot \ln \left(\left(\frac{r_h}{3.7} \right)^{1.11} + \psi_\lambda \right) \right)^{-2}$$

$$\psi_\lambda = \frac{6.9}{Re+Re_t\left(1-tanh\left(\frac{Re}{Re_t}\right)\right)}$$

Reynolds-Number:

$$Re = \frac{\rho \cdot |f| \cdot d_h}{a \cdot \eta}$$

$$Re_t = \frac{2 \cdot 6.9}{1 - \left(\frac{r_h}{3.7} \right)^{1.11}}$$

Physical effect description

The HR element is a hydraulic resistance depending on the pipe friction. The causality parameter describes the dependence of Darcy-Weisbach equation for pressure loss from either volume flow (par_caus = 1) or pressure (par_caus = 2) [1]. The approach used to solve the equation for hydraulic pipe friction factor for laminar and turbulent flow explicitly and continuously is based on the Haaland approximation [3].

Hydraulics

Resistance

HRL



Icon HRL

Classes

| Name | Type | Description |
|--------|-----------|---|
| port_p | connector | port for flow and effort |
| port_n | connector | port for flow and effort |
| hprop | model | properties of fluid, model of viscosity and density |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|---------|-----------------------------------|---|
| temp | parameter | t | Real | K | temperature |
| part_mass_air | parameter | - | Real | - | mass proportion of undissolved air to total mass of mixture |
| e_nom | parameter | - | Real | Pa | nominal effort = nominal pressure |
| f_nom | parameter | - | Real | m^3 / s | nominal flow = nominal volume flow |
| e_ref | parameter | p_{ref} | Real | Pa | reference pressure difference |
| f_ref | parameter | f_{ref} | Real | m^3 / s | reference volume flow |
| eta_ref | parameter | η_{ref} | Real | $\text{kg} / \text{s m}$ | reference viscosity |
| p | parameter | k | Real | - | volume flow exponent for resistance calculation |
| r_min | parameter | - | Real | $\text{kg} / \text{m}^4 \text{s}$ | minimal resistance |
| r_max | parameter | - | Real | $\text{kg} / \text{m}^4 \text{s}$ | maximal resistance |
| par_caus | parameter | - | Integer | - | constitutive equation causality, par_caus = 1(effort out), par_caus = 2(flow out) |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m^3 / s | flow = volume flow |
| rho | variable | ρ | Real | kg / m^3 | fluid density |
| eta | variable | η | Real | $\text{kg} / \text{s m}$ | fluid viscosity |
| r | variable | r | Real | $\text{kg} / \text{m}^4 \text{s}$ | resistance |

Mathematical description

Equation for resistance:

$$p = \frac{p_{ref} \cdot \eta}{\eta_{ref} \cdot f_{ref}^k} \cdot f^k$$

Physical effect description

The HRL element is a hydraulic resistance equivalent to the HR element but only for laminar flow. The causality parameter describes the dependence of the equation for pressure loss from either volume flow (par_caus = 1) or pressure (par_caus = 2) [1].

Hydraulics

Resistance

HRT



Icon HRT

Classes

| Name | Type | Description |
|--------|-----------|---|
| port_p | connector | port for flow and effort |
| port_n | connector | port for flow and effort |
| hprop | model | properties of fluid, model of viscosity and density |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|---------|-----------------------------------|---|
| temp | parameter | t | Real | K | temperature |
| part_mass_air | parameter | - | Real | - | mass proportion of undissolved air to total mass of mixture |
| e_nom | parameter | - | Real | Pa | nominal effort = nominal pressure |
| f_nom | parameter | - | Real | m^3 / s | nominal flow = nominal volume flow |
| e_ref | parameter | p_{ref} | Real | Pa | reference pressure difference |
| f_ref | parameter | f_{ref} | Real | m^3 / s | reference volume flow |
| rho_ref | parameter | ρ_{ref} | Real | $\text{kg} / \text{s m}$ | reference viscosity |
| p | parameter | k | Real | - | volume flow exponent for resistance calculation |
| r_min | parameter | - | Real | $\text{kg} / \text{m}^4 \text{s}$ | minimal resistance |
| r_max | parameter | - | Real | $\text{kg} / \text{m}^4 \text{s}$ | maximal resistance |
| par_caus | parameter | - | Integer | - | constitutive equation causality, par_caus = 1(effort out), par_caus = 2(flow out) |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m^3 / s | flow = volume flow |
| rho | variable | ρ | Real | kg / m^3 | fluid density |
| eta | variable | η | Real | $\text{kg} / \text{s m}$ | fluid viscosity |
| r | variable | r | Real | $\text{kg} / \text{m}^4 \text{s}$ | resistance |

Mathematical description

Equation for resistance:

$$p = \frac{p_{ref} \cdot \rho}{\rho_{ref} \cdot f_{ref}^k} \cdot f^k$$

Physical effect description

The HRT element is a hydraulic resistance equivalent to the HR element but only for turbulent flow. The causality parameter describes the dependence of the equation for pressure loss from either volume flow (par_caus = 1) or pressure (par_caus = 2)[1].

Hydraulics

Resistance

MHRT



Icon MHRT

Classes

| Name | Type | Description |
|---------|-----------|---|
| port_p | connector | port for flow and effort |
| port_n | connector | port for flow and effort |
| hprop | model | properties of fluid, model of viscosity and density |
| port_in | input | input for control signal |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|---------|--------------------------|---|
| temp | parameter | t | Real | K | temperature |
| part_mass_air | parameter | - | Real | - | mass proportion of undissolved air to total mass of mixture |
| e_nom | parameter | - | Real | Pa | nominal effort = nominal pressure |
| f_nom | parameter | - | Real | m^3 / s | nominal flow = nominal volume flow |
| e_ref | parameter | p_{ref} | Real | Pa | reference pressure difference |
| f_ref | parameter | f_{ref} | Real | m^3 / s | reference volume flow |
| rho_ref | parameter | ρ_{ref} | Real | kg / m^3 | reference density |
| p | parameter | k | Real | | volume flow exponent for resistance calculation |
| par_caus | parameter | - | Integer | - | constitutive equation causality, par_caus = 1(effort out), par_caus = 2(flow out) |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m^3 / s | flow = volume flow |
| rho | variable | ρ | Real | kg / m^3 | fluid density |
| eta | variable | η | Real | $\text{kg} / \text{s m}$ | fluid viscosity |
| s_c | variable | s_c | Real | - | control signal |

Mathematical description

Equation for resistance:

$$p = \frac{1}{s_c} \cdot \frac{p_{ref} \cdot \rho}{\rho_{ref} \cdot f_{ref}}^k \cdot f^k$$

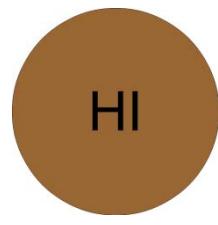
Physical effect description

The MHRT element is a modulated hydraulic resistance for turbulent flow equivalent to the HRT element but with an additional control signal. The causality parameter describes the dependence of the equation for pressure loss from either volume flow (par_caus = 1) or pressure (par_caus = 2) [1].

Hydraulics

Inductance

HI



Icon HI

Classes

| Name | Type | Description |
|--------|-----------|---|
| port_p | connector | port for flow and effort |
| port_n | connector | port for flow and effort |
| hprop | model | properties of fluid, model of viscosity and density |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|------|--------------------------|---|
| temp | parameter | t | Real | K | temperature |
| part_mass_air | parameter | - | Real | - | mass proportion of undissolved air to total mass of mixture |
| e_nom | parameter | - | Real | Pa | nominal effort |
| f_nom | parameter | - | Real | m^3 / s | nominal flow |
| a | parameter | a | Real | m^2 | cross sectional area of pipe |
| l | parameter | l | Real | m | length of pipe |
| f_0 | parameter | f_0 | Real | m^3 / s | initial flow |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m^3 / s | flow = volume flow |
| rho | variable | ρ | Real | kg / m^3 | fluid density |
| eta | variable | η | Real | $\text{kg} / \text{s m}$ | fluid viscosity |

Mathematical description

Equation for inductance:

$$p = \frac{d}{dt} \left(\frac{l}{a} \cdot \rho \cdot f \right)$$

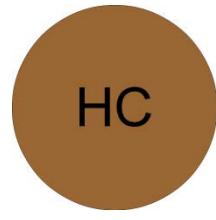
Physical effect description

The HI element is a hydraulic inductance. It describes the pressure difference within a tube that is necessary for a change of volume flow [1].

Hydraulics

Capacitance

HC



Icon HC

Classes

| Name | Type | Description |
|--------|-----------|---|
| port_p | connector | port for flow and effort |
| port_n | connector | port for flow and effort |
| hprop | model | properties of fluid, model of viscosity and density |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|------|--------------------------|---|
| temp | parameter | t | Real | K | temperature |
| part_mass_air | parameter | - | Real | - | mass proportion of undissolved air to total mass of mixture |
| e_nom | parameter | - | Real | Pa | nominal effort |
| f_nom | parameter | - | Real | m^3 / s | nominal flow |
| c | parameter | c | Real | m^2 | compressibility $(dV/dp)/V$ |
| v_0 | parameter | v_0 | Real | m^3 | volume at initial pressure |
| e_0 | parameter | p_0 | Real | Pa | initial pressure |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m^3 / s | flow = volume flow |
| rho | variable | ρ | Real | kg / m^3 | fluid density |
| eta | variable | η | Real | $\text{kg} / \text{s m}$ | fluid viscosity |
| v | variable | v | Real | m^3 | volume of hydraulic node |

Mathematical description

Equation for capacity:

$$f = \frac{1}{\rho} \cdot \frac{d(\rho \cdot v)}{dt}$$

Equation for volume of hydraulic node:

$$v = v_0 \cdot e^{c \cdot (p - p_0)}$$

Physical effect description

The HC element is a hydraulic capacity. It is based on the effect, that a flexible tube could bear additional volume of a liquid in case of an increase in pressure[1].

Hydraulics

Capacitance

MHC



MHC

Icon MHC

Classes

| Name | Type | Description |
|---------|-----------|---|
| port_p | connector | port for flow and effort |
| port_n | connector | port for flow and effort |
| hprop | model | properties of fluid, model of viscosity and density |
| port_in | input | input for volume at initial pressure |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|------|--------------------------|---|
| temp | parameter | t | Real | K | temperature |
| part_mass_air | parameter | - | Real | - | mass proportion of undissolved air to total mass of mixture |
| e_nom | parameter | - | Real | Pa | nominal effort = nominal pressure |
| f_nom | parameter | - | Real | m^3 / s | nominal flow = nominal volume flow |
| c | parameter | c | Real | m^2 | compressibility $(dV/dp)/V$ |
| e_0 | parameter | p_0 | Real | Pa | initial pressure |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m^3 / s | flow = volume flow |
| rho | variable | ρ | Real | kg / m^3 | fluid density |
| eta | variable | η | Real | $\text{kg} / \text{s m}$ | fluid viscosity |
| v | variable | v | Real | m^3 | volume of hydraulic node |
| v_0 | variable | v_0 | Real | m^3 | volume at initial pressure |

Mathematical description

Equation for capacity:

$$f = \frac{1}{\rho} \cdot \frac{d(\rho \cdot v)}{dt}$$

Equation for volume of hydraulic node:

$$v = v_0 \cdot e^{c \cdot (p - p_0)}$$

Physical effect description

The MHC element is a modulated hydraulic capacity equivalent to the HC element but with an additional signal input for the information about the volume at initial pressure [1].

Mechanics

Source

MSe_masy



Icon MSe_masy

Classes

| Name | Type | Description |
|---------|-----------|--------------------------|
| port | connector | port for flow and effort |
| port_in | input | input for control signal |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|--------------|-----------|-------------------|------|--------------------|--|
| par_dyn_1_1 | parameter | - | Real | N m | initial torque, relative value, par_dyn_1_1 in R and par_dyn_1_1 in (0,1) |
| par_dyn_1_2 | parameter | - | Real | N m | torque build-up, time |
| par_dyn_1_3 | parameter | - | Real | N m | torque build-up, relative value, par_dyn_1_3 in R and par_dyn_1_3 in (0,1) |
| par_dyn_2_1 | parameter | - | Real | N m | starting oscillation, torque amplitude |
| par_dyn_2_2 | parameter | - | Real | N m | starting oscillation, damping parameter, par_dyn_2_2 in R and par_dyn_2_2 in (0,1) |
| par_dyn_2_3 | parameter | - | Real | N m | starting oscillation, frequency |
| par_torque_1 | parameter | - | Real | Hz | static torque, start value |
| par_torque_2 | parameter | - | Real | Hz | static torque, saddle value |
| par_torque_3 | parameter | - | Real | Hz | static torque, breakdown value |
| par_torque_4 | parameter | - | Real | Hz | static torque, nominal value |
| par_freq_1 | parameter | - | Real | Hz | rotational frequency, start value |
| par_freq_2 | parameter | - | Real | Hz | rotational frequency, saddle value |
| par_freq_3 | parameter | - | Real | Hz | rotational frequency, breakdown value |
| par_freq_4 | parameter | - | Real | Hz | rotational frequency, nominal value |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m ³ / s | flow = volume flow |
| t_stat | variable | t _{stat} | Real | N m | static torque |
| f_rot | variable | f _{rot} | Real | Hz | rotational frequency |
| u | variable | u | Real | - | input variable, u in R and u in (0,+inf) |
| x_dyn[3] | variable | x _{dyn} | Real | - | dynamic state |

Mathematical description

Equation for effort:

$$e = t_{stat} \cdot x_{dyn}[1] + x_{dyn}[2]$$

Equation for flow:

$$f = 4 \cdot \arcsin(1) * f_{rot}$$

Physical effect description

The MSe_masy element is a mechanical source of effort represented by an asynchronous machine. The model includes the dynamic air gap torque during the starting period of the machine [4].

Mechanics

Source

MSe_masy_stat



MSe
masy

Icon MSe_masy_stat

Classes

| Name | Type | Description |
|---------|-----------|--------------------------|
| port | connector | port for flow and effort |
| port_in | input | input for control signal |

Parameters, variables and constants

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|--------------|-----------|-------------------|------|----------------|--|
| par_torque_1 | parameter | - | Real | N m | static torque, start value |
| par_torque_2 | parameter | - | Real | N m | static torque, saddle value |
| par_torque_3 | parameter | - | Real | N m | static torque, breakdown value |
| par_torque_4 | parameter | - | Real | N m | static torque, nominal value |
| par_freq_1 | parameter | - | Real | Hz | rotational frequency, start value |
| par_freq_2 | parameter | - | Real | Hz | rotational frequency, saddle value |
| par_freq_3 | parameter | - | Real | Hz | rotational frequency, breakdown value |
| par_freq_4 | parameter | - | Real | Hz | rotational frequency, nominal value |
| e | variable | p | Real | Pa | effort = pressure |
| f | variable | f | Real | m ³ | flow = volume flow |
| t_stat | variable | t _{stat} | Real | N m | static torque |
| f_rot | variable | f _{rot} | Real | Hz | rotational frequency |
| u | variable | u | Real | - | input variable, u in R and u in (0,+inf) |

Mathematical description

Equation for effort:

$$e = t_{stat} \cdot u$$

Equation for flow:

$$f = 4 \cdot \arcsin(1) * f_{rot}$$

Physical effect description

The MSe_masy_state element is a mechanical source of effort based on the MSe_masy element. The difference is that the mathematical description of the torque characteristic of the asynchronous machine is based on a static model [4].

Media

Generic

eta_air

Parameters, inputs and outputs

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|------|---------------------------------------|---|
| press | input | p | Real | Pa | pressure |
| temp | input | T | Real | K | temperature |
| eta | output | η | Real | kg / s m | air viscosity |
| press_ref | parameter | p_{ref} | Real | Pa | reference pressure |
| temp_ref | parameter | T_{ref} | Real | T | reference temperature |
| par_eta_press | parameter | C_1 | Real | $\text{Pa}^{-0.01} \text{ K}^{-0.75}$ | viscosity index for pressure dependency of air |
| par_eta_temp | parameter | C_0 | Real | K^2 / Pa | viscosity index for temperature dependency of air |
| eta_ref | parameter | η_{ref} | Real | kg / s m | viscosity at reference conditions of air |

Mathematical description

Equation for viscosity of air:

$$\eta = \eta_{ref} \cdot \left(\frac{C_1 \cdot p^{0.01} \cdot T^{0.75} + C_0 \cdot p \cdot T^{-2}}{C_1 \cdot p_{ref}^{0.01} \cdot T_{ref}^{0.75} + C_0 \cdot p_{ref} \cdot T_{ref}^{-2}} \right)$$

Function description

Eta_air is an approximation function of experimental data for the viscosity of air [5].

Media

Generic

eta_roelands

Parameters, inputs and outputs

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|---------|----------------|------|----------|---|
| press | input | p | Real | Pa | pressure |
| temp | input | T | Real | K | temperature |
| press_ref | input | p_{ref} | Real | Pa | reference pressure |
| temp_ref | input | T_{ref} | Real | T | reference temperature |
| par_eta_press | input | ζ | Real | - | viscosity index for pressure dependency of oil |
| par_eta_temp | input | ξ | Real | - | viscosity index for temperature dependency of oil |
| eta_ref | input | η_{ref} | Real | kg / s m | viscosity at reference conditions of oil |
| eta | output | η | Real | kg / s m | oil viscosity |

Mathematical description

Equation for viscosity of oil:

$$\eta = \eta_{ref} \cdot \exp \left(\ln \left(\frac{\eta_{ref}}{6.315 \cdot 10^{-5} Pa \cdot s} \right) \cdot \psi_{exp} \right)$$

$$\psi_{exp} = -1 + \left(1 + \frac{p - p_{ref}}{1.96 \cdot 10^8 Pa} \right)^{\zeta} \cdot \left(\frac{T - 138 K}{T_{ref} - 138 K} \right)^{-\xi}$$

Function description

Eta_roelands is a function for the viscosity of hydraulic oil developed by Roelands [6].

Media

Generic

eta_mix

Parameters, inputs and outputs

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|--------------|---------|-----------------|------|----------|--|
| parts_vol[:] | input | $C_{parts,vol}$ | Real | - | volume parts of air and oil in mixture |
| eta_parts[n] | input | η_{parts} | Real | kg / s m | viscosity of different volume parts |
| eta | output | η | Real | kg / s m | mixture viscosity |

Mathematical description

Equation for viscosity of mixture:

$$\eta = \frac{\sum_{i=1}^n (\eta_{parts,i} \cdot C_{parts,vol,i}^{2/3})}{\sum_{i=1}^n (C_{parts,vol,i}^{2/3})}$$

Function description

Eta_mix is a function for the viscosity of mixture of oil and air. Index n is the size of the vector parts_vol.

Media

Generic

rho_air

Parameters, inputs and outputs

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|-----------|----------------|------|---------------------|--|
| press | input | p | Real | Pa | pressure |
| temp | input | T | Real | K | temperature |
| rho | output | ρ | Real | kg / m ³ | Density of air |
| press_ref | parameter | p_{ref} | Real | Pa | reference pressure |
| temp_ref | parameter | T_{ref} | Real | T | reference temperature |
| par_rho_press | parameter | n | Real | - | density index for pressure dependency (polytropic exponent) of air |
| par_rho_temp | parameter | m | Real | - | density index for temperature dependency of air |
| rho_ref | parameter | ρ_{ref} | Real | kg / m ³ | density at reference conditions of air |

Mathematical description

Equation for density of air:

$$\rho = \rho_{ref} \cdot \left(\frac{T_{ref}}{T}\right)^m \cdot \left(\frac{p}{p_{ref}}\right)^n$$

Function description

Rho_air is a function for the density of air and is described by the ideal gas law.

Media

Generic

rho_exp

Parameters, inputs and outputs

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|---------|----------------|------|---------------------|---|
| press | input | p | Real | Pa | pressure |
| temp | input | T | Real | K | temperature |
| press_ref | input | p_{ref} | Real | Pa | reference pressure |
| temp_ref | input | T_{ref} | Real | T | reference temperature |
| par_rho_press | input | κ | Real | Pa ⁻¹ | compressibility factor of oil |
| par_rho_temp | input | γ | Real | K ⁻¹ | coefficient of thermal expansion of oil |
| rho_ref | input | ρ_{ref} | Real | kg / m ³ | density at reference conditions of oil |
| rho | output | ρ | Real | kg / m ³ | density of oil |

Mathematical description

Equation for density of oil:

$$\rho = \rho_{ref} \cdot e^{\kappa \cdot (p - p_{ref}) - \gamma \cdot (t - T_{ref})}$$

Function description

Rho_exp is a function for the density of oil based on the exponential relation with compressibility factor and coefficient of thermal expansion.

Media

Generic

rho_mix

Parameters, inputs and outputs

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|---------|------------------|------|---------------------|--------------------------------------|
| parts_mass[:] | input | $c_{parts,mass}$ | Real | - | mass parts of air and oil in mixture |
| rho_parts[n] | input | ρ_{parts} | Real | kg / m ³ | density of different mass parts |
| rho | output | ρ | Real | kg / m ³ | density of mixture |

Mathematical description

Equation for density of mixture:

$$\rho = \frac{1}{\sum_{i=1}^n \left(\frac{c_{parts,mass,i}}{\rho_{parts,i}} \right)}$$

Function description

Rho_mix is a function for the density of mixture of oil and air. Index n is the size of the vector parts_mass.

Media

Generc

parts_vol

Parameters, inputs and outputs

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|---------------|---------|------------------|------|---------|--------------------------------------|
| parts_mass[:] | input | $c_{parts,mass}$ | Real | kg | mass parts of air and oil in mixture |
| rho_parts[xn] | input | ρ_{parts} | Real | kg / m3 | density of different volume parts |
| parts[n] | output | $c_{parts,vol}$ | Real | - | volume parts of mass components |

Mathematical description

Equation for parts of volume:

$$c_{parts,vol} = \frac{\frac{c_{parts,mass}}{\rho_{parts}}}{\sum_{i=1}^n \left(\frac{c_{parts,mass,i}}{\rho_{parts,i}} \right)}$$

Function description

Parts_vol is a function for the volume parts of mass components in the mixture. Index n is the size of the vector parts_mass.

Media

Generic

press_cav

Parameters, inputs and outputs

| Name | Element | Formula Symbol | Type | SI-Unit | Description |
|--------------------|---------|----------------|------|----------|---|
| temp | input | T | Real | K | temperature |
| temp_ref | input | T_{ref} | Real | K | reference temperature |
| par_press_cav_temp | input | C_0 | Real | K^{-1} | temperature index for cavitation pressure dependency of oil |
| press_cav_ref | input | $p_{cav,ref}$ | Real | Pa | reference cavitation pressure |
| press | input | p | Real | Pa | cavitation pressure |

Mathematical description

$$\text{Equation for cavitation pressure: } p = p_{cav,ref} \cdot \left(\frac{T}{T_{ref}} \right) \cdot e^{C_0 \cdot (T - T_{ref})}$$

Function description

Press_cav is an approximation function of experimental data for the cavitation pressure of liquid fluid.

References

- [1] H. Murrenhoff, Grundlagen der Fluidtechnik - Teil 1: Hydraulik, Aachen: Shaker Verlag, 2012.
- [2] T. Krist, Hydraulik Fluidtechnik, Würzburg: Vogel-Verlag, 1987.
- [3] S. E. Haaland, "Simple and explicit formulas for the friction factor in turbulent pipe flow," *Transactions of the ASME, Journal of Fluids Engineering*, vol. 105, no. 1, pp. 89-90, 1983.
- [4] A. Laschet, Simulation von Antriebssystemen, Berlin Heidelberg New York: Springer-Verlag, 1988.
- [5] J. H. Spurk and N. Aksel, Strömungslehre - Einführung in die Theorie der Strömungen, Berlin Heidelberg New York: Springer-Verlag, 2006.
- [6] C. J. A. Roelands and J. C. Vlugter, "The Viscosity-Temperature-Pressure Relationship of Lubricating Oils and Its Correlation Whith Chemical Constitution.,," *ASME Journal of Basic Engineering*, vol. 11, pp. 601-611, 1963.