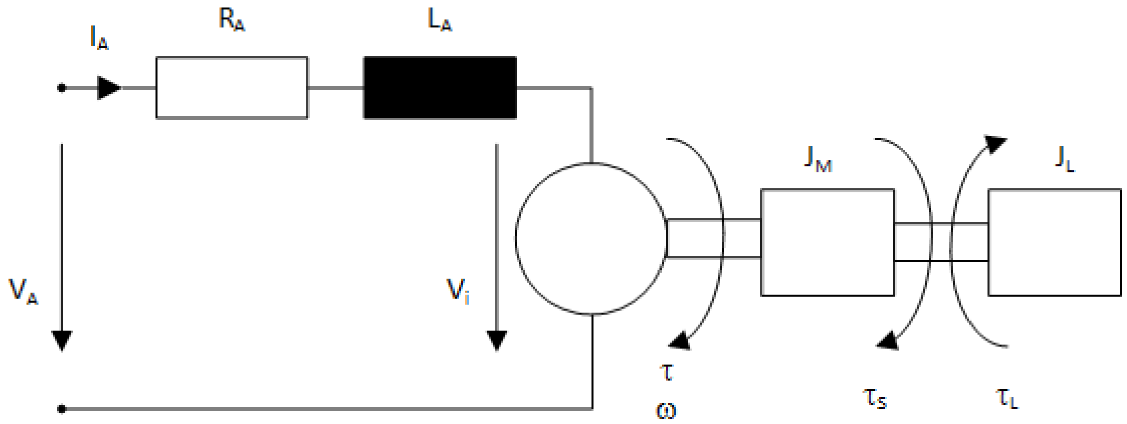
The kinetic energy of rotating masses is converted to electric energy using an electric machine and subsequently dissipated in the internal resistance and the external braking resistance to heat.

**<u>DC-Machine with Permanentmagnet Excitation</u>**




Farady's Law: $V_i = k\Phi \cdot \omega$

Lorentz' Law: $\tau = k\Phi \cdot I_A$

These two basic equations get implemented in the „electro-mechanical converter".

Kirchhoff's voltage law: $V_A = R_A \cdot I_A + L_A \cdot \frac{dI_A}{dt} + V_i$

Equation of Motion: $\tau - \tau_L = (J_r + J_L) \cdot \frac{d\omega}{dt}$

**<u>Parameter sets</u>**:

| | | | Unite 48V | Unite XL |
|---|---|---|---|---|
| Nominal armature voltage | $V_{ANom}$ | V | 48 | 480 |
| Nominal armature current | $I_{ANom}$ | A | 20 | 10 |
| Armature resistance @ 20°C | $R_{ARef}$ | Ω | 0,23184 | 2,625 |
| Nominal armature temperature | $T_{ANom}$ | °C | 95 | 95 |
| Armature inductance | $L_A$ | mH | 0,6 | 6,5 |
| Induced voltage at nom. speed | $V_{iNom}$ | V | 42 | 446 |
| Nominal shaft speed | $\omega_{Nom}$ | rpm | 3150 | 1500 |
| No-load speed | $\omega_0$ | rpm | 3600 | 1614 |
| Machine constant | $k\phi$ | Wb | 0,127322243 | 2,839511426 |
| Nominal torque | $\tau_{Nom}$ | Nm | 2,546444851 | 28,39511426 |
| Rotor's moment of inertia | $J_r$ | kg.m² | 0,0012 | 0,012 |
| Stator's moment of inertia | $J_s$ | | 4 x $J_r$ | 4 x $J_r$ |

$$R_A(95°) = R_A(20°) \cdot \frac{235° + 95°}{235° + 20°}$$

$$V_{iNom} = V_{ANom} - R_{A95°} \cdot I_{ANom} = k\phi \cdot \omega_{Nom}$$

$$V_{ANom} = k\phi \cdot \omega_0 \rightarrow \omega_0 = \omega_{Nom} \cdot \frac{V_{ANom}}{V_{iNom}}$$

Analytic solution:

Looking at the above mentioned equation system: A rapid change of current only occurs immediately after shorting the induced voltage. Afterwards the current follows the change of induced voltage, which in turn is determined by mechanical speed, whose change is comparably low. Therefore we neglect $\frac{dI_A}{dt}$.

$$I_a = -\frac{k\phi}{R_a + R_b} \cdot \omega$$
$$\tau = -\frac{k\phi^2}{R_a + R_b} \cdot \omega$$

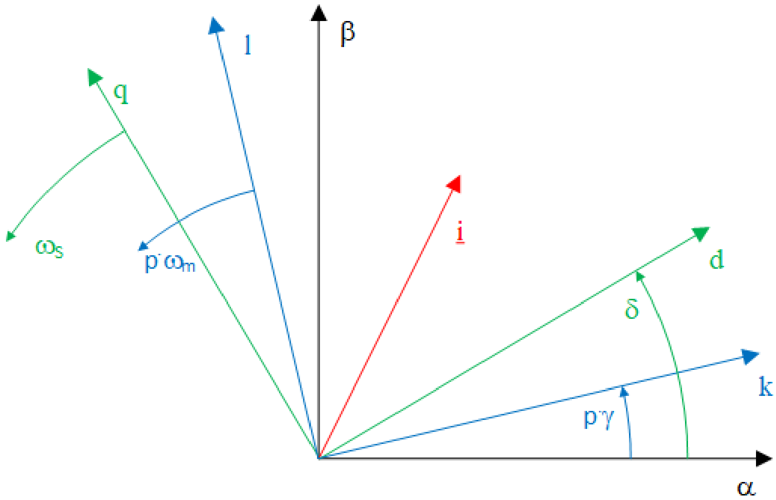
The braking torque $\tau$ is proportional to angular velocity $\omega$, to the square of magnetic flux $\phi$ and to geometry $k$ as well as indirect proportional to the resistance $R_a + R_b$.

$$J \cdot \frac{d\omega}{dt} = -\frac{k\phi^2}{R_a + R_b} \cdot \omega$$

$J$ is the sum of inertias of the machine's rotor and the load.
Separation of variables:

$$\frac{d\omega}{\omega} = -\frac{k\phi^2}{J \cdot (R_a + R_b)} \cdot dt$$
$$ln(\omega) = ln(\omega_0) - \frac{k\phi^2}{J \cdot (R_a + R_b)} \cdot t$$
$$\tau = \frac{J \cdot (R_a + R_b)}{k\phi^2}$$
$$\omega = \omega_0 \cdot e^{-\frac{t}{\tau}}$$

**Synchronous Machine with Permanentmagnet Excitation**




Stator fixed frame $\alpha + j\beta$

Rotor fixed frame $k + jl$

Field fixed frame $d + jq$

For synchronous machines the field fixed frame and the rotor fixed frame are the same.

$\gamma$ is the mechanical angle of the rotor $\varphi_m$ (with respect to the stator).

For this machine Clarke and Park transform for voltages and currents are implemented in the corresponding electro-mechanical converter:

$$\begin{bmatrix} cos(p\cdot\varphi_m) & -sin(p\cdot\varphi_m) \\ sin(p\cdot\varphi_m) & cos(p\cdot\varphi_m) \end{bmatrix}\cdot\begin{bmatrix} i_d \\ i_q \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2}{3}\cdot\begin{bmatrix} cos\left(0\cdot\frac{2\pi}{3}\right) & cos\left(1\cdot\frac{2\pi}{3}\right) & cos\left(2\cdot\frac{2\pi}{3}\right) \\ sin\left(0\cdot\frac{2\pi}{3}\right) & sin\left(1\cdot\frac{2\pi}{3}\right) & sin\left(2\cdot\frac{2\pi}{3}\right) \end{bmatrix}\cdot\begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix}$$

The zero components fulfills the following equation:

$$i_0 = \frac{1}{3}\cdot(i_u + i_v + i_w)$$

$$v_0 = L_0\cdot\frac{di_0}{dt}$$

**Set of Equations**:

Due to magnetic saliency and the permanent magnets acting in one foxed direction with respect to the rotor the equations have to be implemented in the rotor fixed frame:

$$v_d = R_S\cdot i_d + L_d\cdot\frac{di_d}{dt} - p\cdot\omega_m\cdot L_q\cdot i_q$$

$$v_q = R_S\cdot i_q + L_q\cdot\frac{di_q}{dt} + p\cdot\omega_m\cdot L_d\cdot i_d + p\cdot\omega_m\cdot\psi_{PM}$$

$$\tau = \frac{3}{2}\cdot p\cdot[\psi_{PM} + (L_d - L_q)\cdot i_d]\cdot i_q$$

$$\tau - \tau_L = (J_r + J_L)\cdot\frac{d\omega}{dt}$$

**Parameter sets**:

| | | | S1FT7102 | S1FT7105 |
|---|---|---|---|---|
| Nominal RMS stator voltage per phase | $V_{sNom}$ | V | 400/√3 | 400/√3 |
| Nominal RMS stator current per phase | $I_{ANom}$ | A | 8 | 15 |
| Stator resistance @ 20°C | $R_{sRef}$ | Ω | 0,6 | 0,15 |
| Nominal stator winding temperature | $T_{sNom}$ | °C | 95 | 95 |
| d-axis inductance | $L_d$ | mH | 12,5 | 4,2 |
| q-axis inductance | $L_q$ | mH | 12,5 | 4,2 |
| zero component inductance | $L_0$ | mH | 1,25 | 0,42 |
| Open circuit RMS voltage at nom. speed | $V_{sOC}$ | V | 183,8571932 | 199,7631931 |
| Nominal shaft speed | $\omega_{Nom}$ | rpm | 1500 | 2000 |
| Number of pole pairs | p | | 5 | 5 |
| Nominal stator frequency | $f_s$ | Hz | 125 | 166,6666667 |
| Rotor's moment of inertia | $J_r$ | kg.m² | 0,009 | 0,018 |
| Stator's moment of inertia | $J_s$ | | 4 x $J_r$ | 4 x $J_r$ |

The machines have $m = 3$ stator phases, the windings are star-connected.

Stator frequency is strictly related to mechanical shaft speed:

$$2\pi \cdot f_s = p \cdot \omega_m$$

Flux linkage space phasor of stator due to permanent magnets can be calculated from open circuit voltage at nominal speed:

$$\sqrt{2} \cdot V_{sOC} = \psi_{PM} \cdot 2\pi \cdot f_s$$

Analytic solution:

Looking at the above mentioned equation system: A rapid change of current only occurs immediately after shorting the induced voltage. Afterwards the current follows the change of induced voltage, which in turn is determined by mechanical speed, whose change is comparably low. Therefore we neglect $\frac{di}{dt}$.

$$i_d = \frac{\omega_S \cdot L_q}{R_S + R_b} \cdot i_q$$
$$i_q \cdot \left(1 + \frac{{\omega_S}^2 \cdot L_d \cdot L_q}{(R_S + R_b)^2}\right) = -\frac{\omega_S \cdot \psi_{PM}}{R_S + R_b}$$
$$\omega_K = \frac{R_S + R_b}{\sqrt{L_d \cdot L_q}}$$
$$\tau = -\frac{3}{2} \cdot p \cdot \frac{{\psi_{PM}}^2}{R_S + R_b} \cdot \omega_K \cdot \frac{1}{\frac{\omega_K}{\omega_S} + \frac{\omega_S}{\omega_K}} = -\frac{3}{2} \cdot p \cdot \frac{{\psi_{PM}}^2}{\sqrt{L_d \cdot L_q}} \cdot \frac{1}{\frac{\omega_K}{\omega_S} + \frac{\omega_S}{\omega_K}}$$

The braking torque $\tau$ follows the same equation as Kloss' formula (for the induction machine using the slip). The maximum torque $\tau_K$ occurs at $\omega_S = \omega_K$:

$$\tau_K = -\frac{3}{4} \cdot p \cdot \frac{{\psi_{PM}}^2}{\sqrt{L_d \cdot L_q}}$$
$$\tau = \frac{2 \cdot \tau_K}{\frac{\omega_K}{\omega_S} + \frac{\omega_S}{\omega_K}}$$

For small speed $\omega_S \ll \omega_K$ we can approximate torque with a linear dependency on speed:

$$\tau = 2 \cdot \tau_K \cdot \frac{\omega_S}{\omega_K}$$

For large speed $\omega_S \gg \omega_K$ we can approximate torque with a hyperbolic dependency on speed:

$$\tau = 2 \cdot \tau_K \cdot \frac{\omega_K}{\omega_S}$$

The equation of motion can be expressed as follows:

$$\frac{J}{p} \cdot \frac{d\omega_S}{dt} = \frac{2 \cdot \tau_K}{\frac{\omega_K}{\omega_S} + \frac{\omega_S}{\omega_K}}$$

$J$ is the sum of inertias of the machine's rotor and the load.
Separation of variables:

$$\left(\frac{\omega_K}{\omega_S} + \frac{\omega_S}{\omega_K}\right) \cdot d\,\frac{\omega_S}{\omega_K} = p \cdot \frac{2 \cdot \tau_K}{J \cdot \omega_K} \cdot dt$$
$$ln\left(\frac{\omega_S}{\omega_K}\right) + \frac{1}{2} \cdot \left(\frac{\omega_S}{\omega_K}\right)^2 + K = p \cdot \frac{2 \cdot \tau_K}{J \cdot \omega_K} \cdot t$$

This term cannot be solved analytically for $\omega_S = p \cdot \omega_m$.